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Large momenta fluctuations of charm quarks in the Quark-Gluon Plasma*

S. Terranova[a], D.M. Zhou[b], and A. Bonasera[c]

Laboratorio Nazionale del Sud, Istituto Nazionale di Fisica Nucleare, Via S. Sofia 44, I-95123 Catania, Italy

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Abstract. We show that large fluctuations of D-mesons kinetic-energy (or momentum) distributions might be a signature of a phase transition to the Quark-Gluon Plasma (QGP). In particular, a jump in the variance of the momenta or kinetic energy, as a function of a control parameter (temperature or Fermi energy at finite baryon densities) might be a signature for a first-order phase transition to the QGP. This behavior is completely consistent with the order parameter defined for a system of interacting quarks both at zero temperature (and finite baryon densities) or at finite temperatures which shows a jump in correspondence with a first-order phase transition to the QGP. The J/Ψ displays exactly the same behavior of the order parameter and of the variance of the D-mesons. We discuss implications for relativistic heavy-ion collisions within the framework of a transport model and possible hints for experimental search.

PACS. 12.39.Pn Potential models – 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes

The production of a new state of matter, the Quark-Gluon Plasma (QGP), can be obtained through ultra-relativistic heavy-ion collisions (RHIC) at CERN and at Brookhaven [1]. The QGP can be formed in the first stages of the collisions, and can be studied through the particles produced.

Some features of the quark matter can be revealed by studying the properties of hadrons in a dense medium. The particle J/Ψ is a good candidate because the formation of the QGP might lead to its suppression [2]. Here we want to show that in reality information about the QGP is carried by the charm quarks. These quarks interact strongly with the unconfined surrounding matter and, as a result, we have a suppression of the J/Ψ, but also large fluctuations of the charm quarks kinetic energies which could be revealed by the D-mesons distributions or other charmed mesons or baryons. To see this, we elaborate on a semiclassical model which has an EOS resembling the well-known properties of nuclear matter and its transition to the QGP at zero temperature and finite baryon densities already discussed in [3]. We simulate the nuclear matter which is composed of nucleons (which are by themselves composite three-quark objects) and its dissolution into quark matter. In addition, for our system of colored quarks, we will show how the color screening is related to the lifetime of the particle J/Ψ in the medium. In particular, we will see that the lifetime of the J/Ψ as a function of density behaves as an order parameter. On exactly the same ground we show that the variances of the charm quarks are large and they "jump" at the critical point for a first-order phase transition. Thus, this quantity, similarly to the lifetime of the J/Ψ, behaves exactly as an order parameter and could give important information about not only the transition to the QGP but also to the order of the phase transition, *i.e.* first, second order or simply crossover to the QGP. Since in our model the phase transition is due to a breaking of a symmetry in color space [3], it does not matter if the control parameter is the baryon density or the temperature. In fact, we have performed calculations at zero baryon density and finite temperatures (which should be a condition closer to RHIC experiments) and found exactly the same behavior for the quantities discussed above. Since D-particles are the lightest charmed mesons, they are produced in heavy-ion collisions at the first stages of the reaction and they have a long mean free path in hadronic matter, thus they are the best probes for the phase transition. Other particles, such as pions and kaons could give information about a possible phase transition as well [4]. However, because of their light masses they can be also produced at later stages of the reaction, *i.e.* in the hadronic stage [5], and/or collide with other particles. Thus, the (large) initial fluctuations might be washed

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[a] e-mail: terranova@lns.infn.it
[b] e-mail: zhou@lns.infn.it
[c] e-mail: bonasera@lns.infn.it

away in the later stages of the collision. In other words, ideal probes for a phase transition are particles with a long mean free path in hadronic matter and rather massive such that they are most probably produced in the early stages of the reaction. Charmed mesons could be such particles at RHIC energies, while at LHC, for instance, one might look for fluctuations of bottom mesons. At GSI/AGS energies kaons might be good probes of the phase transition.

An important ingredient of our approach is a constraint to satisfy the Pauli principle. The approach, dubbed Constrained Molecular Dynamics (CoMD) has been successfully applied to relativistic and non-relativistic [6,7] heavy-ion collisions and plasma physics as well [8]. On the other hand, for systems at high temperatures the Pauli principle does not play an important role and calculations may be easily extended to that case.

The color degrees of freedom of quarks are taken into account through the Gell-Mann matrices and their dynamics is solved classically, in phase space, following the evolution of the distribution function. The quarks interact through Richardson's potential $V(\mathbf{r}_i, \mathbf{r}_j)$:

$$V(\mathbf{r}_{i,j}) = 3\sum_{a=1}^{8} \frac{\lambda_i^a}{2}\frac{\lambda_j^a}{2}\left[\frac{8\pi}{33-2n_f}\Lambda\left(\Lambda r_{ij} - \frac{f(\Lambda r_{ij})}{\Lambda r_{ij}}\right)\right] \quad (1)$$

and [9]

$$f(t) = 1 - 4\int \frac{\mathrm{d}q}{q}\frac{e^{-qt}}{[\ln(q^2-1)]^2+\pi^2}. \quad (2)$$

λ^a are the Gell-Mann matrices. We fix the number of flavors $n_f = 2$ and the parameter $\Lambda = 0.25$ GeV, ($\hbar, c = 1$). Here we assume the potential to be dependent on the relative coordinates only. The first term is the linear term, responsible for the confinement, the second is the Coulomb term [9]. We solve Hamilton's classical equations.

Initially we distribute randomly the quarks in a box of side L in coordinate space and in a sphere of radius p_f in momentum space. p_f is the Fermi momentum estimated in a simple Fermi-gas model by imposing that a cell in phase space of size $h = 2\pi$ can accommodate at most g_q identical quarks of different spins, flavors and colors. $g_q = n_f \times n_c \times n_s$ is the degeneracy number, n_c is the number of colors (three different colors are used: red, green and blue) hence $n_c = 3$; $n_s = 2$ is the number of spins [1].

A simple estimate gives the following relation between the density of quarks of flavor f with colors, ρ_{qfc}, and the Fermi momentum:

$$\rho_{qfc} = \frac{3n_s}{6\pi^2}p_f^3\,. \quad (3)$$

We generate many events and take the average over all events in each cell on the phase space. For each particle we calculate the average occupation, *i.e.* the probability that a cell in the phase space is occupied. To describe the Fermionic nature of the system we impose that the average occupation for each particle is less than or equal to 1 ($\bar{f}_i \leq 1$). This is accomplished through the use of constraints and its numerical implementation is discussed in detail in refs. [7,3] and more recently using the Lagrange multiplier method in ref. [8]. Here we illustrate the case with $m_u = 5$ MeV, $m_d = 10$ MeV and using a cut-off to the linear term of 2 fm [3] which has been introduced to avoid numerical uncertainties. Such a system displays a clear first-order phase transition at high baryon densities. The results are completely analogous for the other parameter sets discussed in [3].

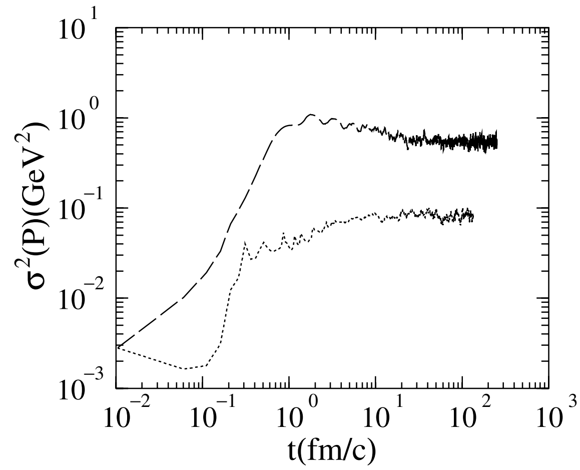

Fig. 1. Time evolution of momentum variances of charm quarks at two densities, above and below the QGP phase transition.

After our system of u and d quarks has evolved to its equilibrium configuration, we insert one J/Ψ particle, *i.e.* $c\bar{c}$-quarks, and let them evolve. The Pauli principle is responsible for the kinetic energy of the light quarks. The Fermi motion increases for increasing densities. On the other hand the embedded charm quarks are not influenced by the Pauli principle because they are different fermions. However, there is a strong interaction among all the quarks given by the Richardson potential. Because of such interaction, the charm quarks start to exchange energy with the surrounding medium and finally get in equilibrium with the other quarks. Thus, c-quarks are a perfect probe of the system since they can wander anywhere in the available phase space of the system. In particular, we can calculate the variance, for instance, in momentum space of the charm quarks. For a non-interacting Fermi gas, the variance in momentum space is given by

$$\sigma^2 = \langle p^2\rangle - \langle p\rangle^2 = \frac{3}{80}p_f^2\,. \quad (4)$$

Similar relations can be obtained for kinetic-energy fluctuations. Of course, because of the interaction, the fluctuations are modified and might spectacularly increase near a phase transition.

In fig. 1, we plot the variances in momentum space *vs.* time for two densities above and below the critical point for a first-order phase transition [3]. One immediately sees that the variances are much larger than our estimate for the Fermi gas given above (respectively $\sigma^2 = 0.0055$ and $0.0328\,\mathrm{GeV}^2$ for the cases displayed in fig. 1) because of the

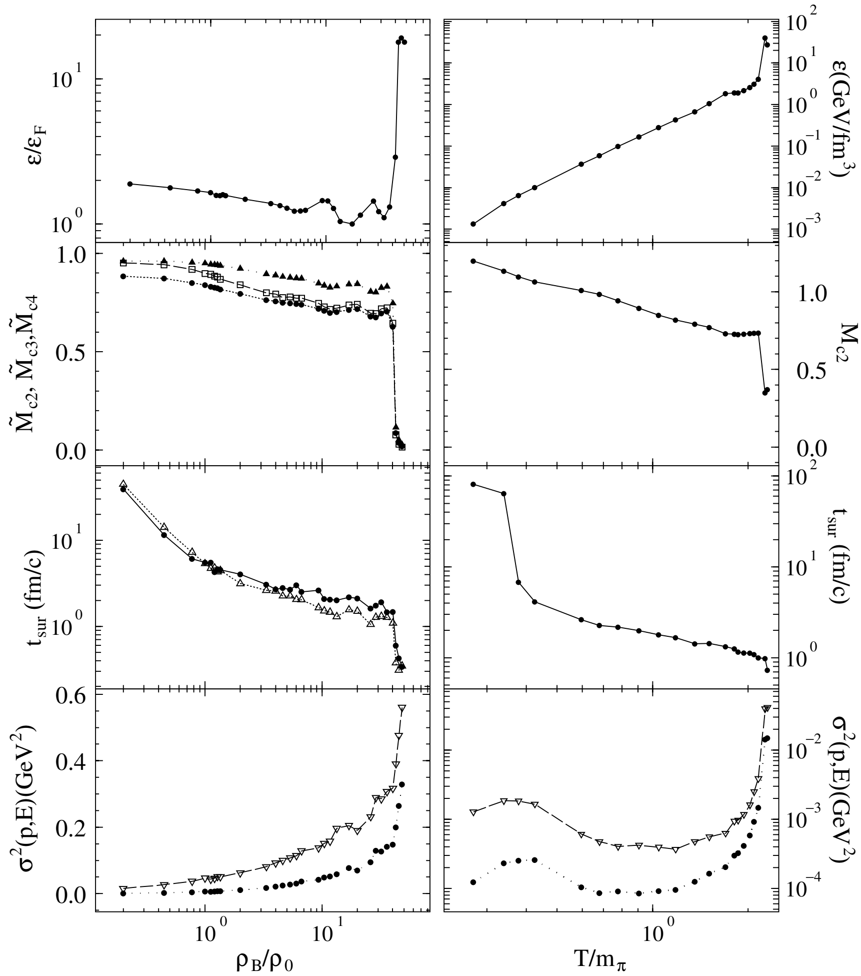

Fig. 2. Energy density (top panels), normalized order parameters (2nd panels and see text), time survival of J/Ψ (3rd panels) and variances in momentum and kinetic energy of c-quarks (bottom panels) *vs.* density divided by the normal density ρ_0 (left panels) or temperature divided by the pion mass (right panels), for $m_u = 5$ MeV, $m_d = 10$ MeV and cut-off = 2 fm.

strong interaction. Initially the variances of the c-quarks are very small, but after a transient time, fluctuations are transferred from the light to the heavy quarks up to a stationary value. As expected, the variances are larger above the phase transition, *i.e.* at high density.

This is clearly demonstrated in fig. 2 (left panels) where the energy density, the order parameters for the 2, 3 and 4 closest particles $\widetilde{M_{c2}}$, $\widetilde{M_{c3}}$, $\widetilde{M_{c4}}$, the J/Ψ lifetime [3] and the variances are plotted *vs.* density. All quantities jump at the critical point [3] signaling a first-order phase transition. In order to show that these properties are independent of the details of the forces and that the charm quarks are real good probes of the phase transition, we have arbitrarily increased of a factor 2 the interaction strength between the $c\bar{c}$-quarks alone. This results in a change of the J/Ψ lifetime (squares in fig. 2, left panels) but the jump of its lifetime at the critical point remains.

These features remain if instead of the baryon density, temperature is the control parameter. In fact, we have performed calculations for a system of interacting quarks and antiquarks at finite temperatures and under the constraint of equal chemical potential, *i.e.* $\mu_q = \mu_{\bar{q}} = 0$, (fig. 2, right panels). In this case the relation between quark (antiquark) density and temperature is given by Wong in ref. [1] (eq. (5), pag. 166). Now the relevant order parameter is $\widetilde{M_{c2}}$ for two quarks (second panel) which takes the value of 4/3 for a pion gas, and becomes 2/3 in the QGP. These limits are obtained at small temperatures (pion gas) and at about 230 MeV temperature where the QGP is formed. The latter value is slightly larger than the one obtained in lattice QCD calculations (about 170 MeV) [10]. However, we did not try to fit the parameter set which is the same used for the zero-temperature case. Notice that the J/Ψ lifetime and the D-meson variances behave similarly to the order parameter, and in particular they have a discontinuity where the energy density jumps. The difference between the finite-temperature case and the degenerate Fermi case is the possibility of a given quark to combine in different ways to quarks and antiquarks because of the color charge. This results in a richness of combinations in the finite-temperature case and it will be discussed in more detail in following works. For the purpose of this

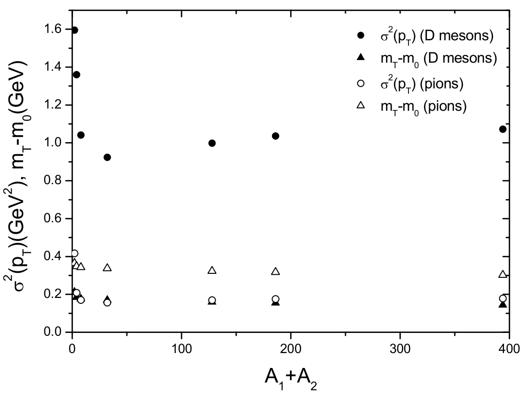

Fig. 3. Variances (circles) and transverse minus rest mass (triangles) *vs.* mass number of the colliding nuclei. Full symbols refer to D-meson production, while open symbols refer to pions.

paper, we only want to stress that indeed the quantity we propose (*i.e.* momentum fluctuations of the D-mesons) is sensitive to a phase transition and to its order as well.

In order to simulate a realistic heavy-ion collisions we have performed some calculations in a transport model at $\sqrt{s} = 200$ GeV. We have used the JPCIAE code which includes Pythia as a generator of elementary collisions [11] and performed calculations for symmetric nucleus-nucleus collisions starting from pp to AuAu. Variances in the transverse momenta of produced open charms are calculated for central collisions (impact parameter $b = 0$ fm). In fig. 3 we plot the variances *vs.* the sum of the mass numbers of the colliding nuclei. The variances are calculated averaging over the events which contain at least one D-meson produced (circles). In the same graph we have plotted the transverse mass minus the rest mass of the particle (triangles), which gives an indication of the degree of equilibration reached. The full symbols refer to D-mesons while the open symbols refer to pions. As expected, such variances are roughly constant, *i.e.* independent of the mass number for colliding ions heavier than oxygen. In fact, this is an expected result since the model contains in principle no phase transition to the QGP. Thus, normalizing the variance say in Au + Au with respect to $p + p$ collisions at the same beam energy should give about 1. An anomalous increase from such a value should indicate that much more physics than contained in our transport code is indeed at play. Furthermore, some anomaly in the variances with increasing mass number of the colliding nuclei should give a clear indication of the occurrence of the phase transition and, possibly, of its order. This is so because we expect no QGP for small systems, while a transition should occur for relatively large nuclei. How large those nuclei should be at RHIC beam energy could be obtained from the analysis we propose in fig. 3. In the same figure we have plotted the average transverse kinetic energy for open charms and pions. Such a quantity should give an indication of the degree of equilibration of the system. As can be easily seen, the latter quantity has a similar behavior of the variance which is what we expect if the system is equilibrated. Indeed one can see that there is a proportionality between the two quantities especially for colliding nuclei heavier than oxygen; furthermore the average kinetic energies for different particles are very close, which is another indication of thermal equilibrium. Looking at the pions results only, it seems that equilibration is reached already for small systems such as $d + d$. This is so because many pions are produced in an event and also pions are produced in different steps of the collision. This results in variances smaller than the one obtained for D-mesons already in our kinetic approach. Thus, pions could give an indication on the temperature reached in the reaction, while a phase transition could be signaled by the possible J/Ψ suppression or the large fluctuations of the D-meson which might be a stronger signal since they are more abundantly produced in the collisions. In this context, large fluctuations are understood compared to the pp case. We have to stress as well that finite-size effects, ambiguities in the impact parameter selections and other dynamical effects could modify our findings. In particular, a jump due to a first-order phase transition may be smoothed by finite-size effects. However, since the number of produced particles is relatively large, we still expect to see some effects of the phase transition. For instance, claims for a first-order phase transition, from liquid to gas, are reported in [12] for heavy-ion collisions below 100 MeV/A, *i.e.* for a system size even smaller than the ones obtained at RHIC/LHC energies.

In conclusion, in this work we have discussed a semiclassical molecular-dynamics approach to infinite matter at finite baryon densities and zero temperature starting from a phenomenological potential that describes the interaction between quarks with color. We have studied the case of a set of parameters which displays a first-order phase transition at high baryon densities. We have shown that, similarly to the order parameter and the J/Ψ lifetime, the variances in momentum space of charm quarks "jump" at the critical point of the phase transition. We have found the same features for a system at high temperatures and zero baryon density. We have proposed an experimental search at RHIC inspired by the CoMD results. In particular we have simulated heavy-ion collisions in a transport model (which does not include a phase transition), at fixed beam energy and changing the mass numbers of the colliding system. We have shown that already for small colliding nuclei such as oxygen, the system could reach thermal equilibrium. An anomalous behavior of the variances of the open-charms transverse momenta as a function of the mass number of the colliding system should give a signal for the transition to the QGP. In particular, variances for pp collisions should be much smaller than those obtained in Au + Au collisions at the same beam energy if there is a phase transition. If not, we expect variances to be in agreement to our estimate in fig. 3. If the large fluctuations are found in open-charms production, then the phase transition could be further studied by changing the beam energy and/or the impact parameter selection for a given system and energy, since we expect a phase transition for central collisions and not for the most peripheral ones.

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